Review Session PHYS-201(d)

January 17th 2025

Grundler

General comments:
(1) “cheat sheet”

I was wondering if we were allowed to print out the math toolbox with all the equations, or if we had to write them down in our formulaire.

Write down the key expressions that you were trained on and which you prefer to have ready. The number of equations depends on the student’s mathematical expertise. If a very special equation was needed we provided it.

General comments:
(2) Problems from other sources than PHYS-201(d)

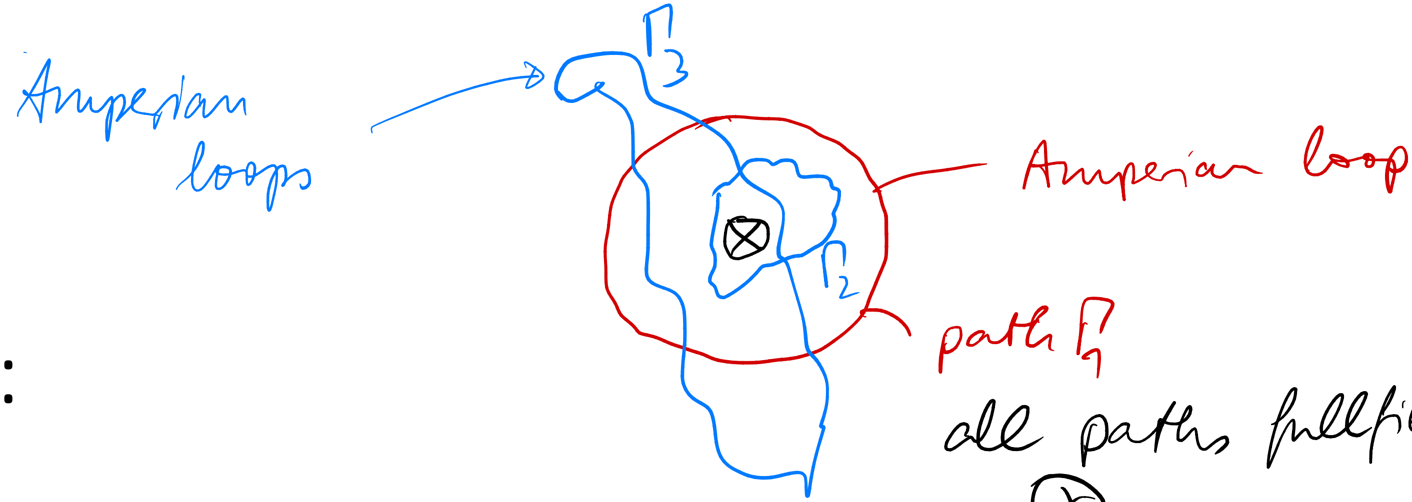
Answers will not be given here in view of the restricted time and to stay in the context of the course.

1. Question:

in exercise 2 of the problem set 9, there is an infinite solenoid and we are asked to find the expression for A.
By analyzing the symmety I recognize that there is a rotational as well as translational symmetry.
But what does that mean for A? :

In which direction does A point relative to B?
And when I write down B=∇×A by using the cylindrical coordinates expression in the math tool box,
how do I know which components cancel?
What confuses me the most is that the solenoid is radially symmetric,
but then why do we find A = A_ϕ*eϕ ?

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{j}(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x' \qquad d^3x' = dV$$



all paths fullfill $\circledast$

2. Question:

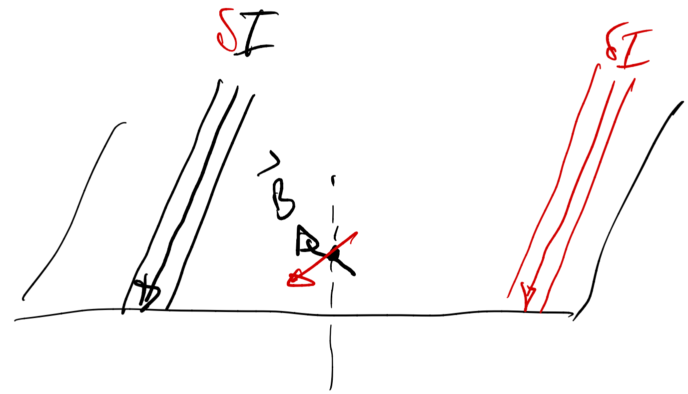
For chapter 4, I'm little bit confused concerning the use of Biot-Savart Law or Ampere Law.
In what case we can use one and what case the other ?
Which is the more simple for which occasion ?

Gauss's law $\oiint \vec{E}\, d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

$$d\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} I \frac{\hat{u}_t \times \vec{u}_r}{r^2}\, dl$$

Ampère's law $\oint \vec{B}\, d\vec{s} = \mu_0 I_{enc}$ $\circledast$

3. Question:



Exercise Sheet 8, Exercise 5 :

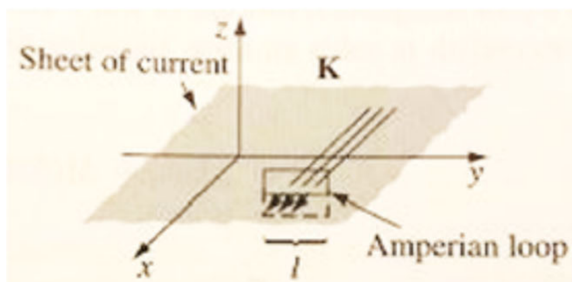
Why i enclosed = K l ?

Why the integral of B over the loop is equal to (2l)B and not 4l ?

~~(4l because the perimeter of the loop is equal to 4l)~~

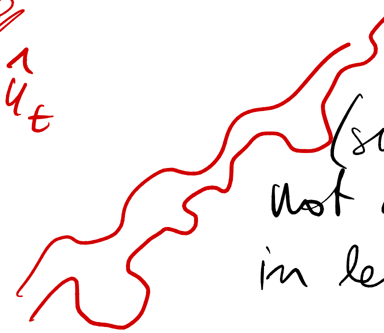
More generally, when we do an integral over a loop,

do we integrate over a surface or a ligne ? I'm confused about the notation integral of a closed loop



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Figure 2: Sheet of current and possible Amperian loop to analyze the magnetic field B after symmetry analysis.



4. Question:

The next question are about the Exercise Sheet 10 :

Exercise 1 : I don't know where come from the expression F = IBL (or F = I (LxB)) and in what configuration did I have to use this one ?

Summary 4.4

Magnetic force: - on an **individual charge *q*** $\boxed{\overrightarrow{F_M} = q\vec{v} \times \vec{B}}$

- on a **current-carrying wire** $\boxed{\vec{F}_M = I \int_{\substack{path\ along \\ the\ length\ of\ wire}} \vec{u}_t \times \vec{B}(x,y,z)dl}$

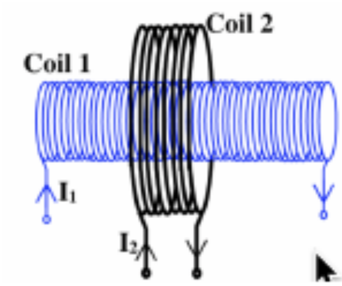
$\vec{u}_t$: unit vector parallel to $\vec{j}$

Rectilinear (straight) wire of length L in uniform field $\boldsymbol{B}_0$: $\boxed{\vec{F}_M = I \cdot \vec{L} \times \vec{B}_0}$ straight wire, homogeneous field B_0, length L in B_0

5. Question (Problem sheet 10):

········Exercise·4·:·What·can·change·in·our·reasoning·if·there·is·i2·in·the·coil·2·?·And·how·we·can· determine·if·there·is·i2·or·not·?·¶

c) Now consider coil 2 to be present. It has a total of N_2 turns. What is the mutual inductance M when one assumes that all the flux from the solenoid (coil 1) passes through the outer coil 2; how does it depend on the parameters n_1, N_2, and R_1? (Hint: No current flow in coil 2, ends of coil 2 are open. To calculate M one must consider that coil 2 encircles the flux of coil 1 N_2 times.)



This was a general image
Taken from some book. Not fine-tuned for the problem set. We create specific images for the written exam.

6. Question:

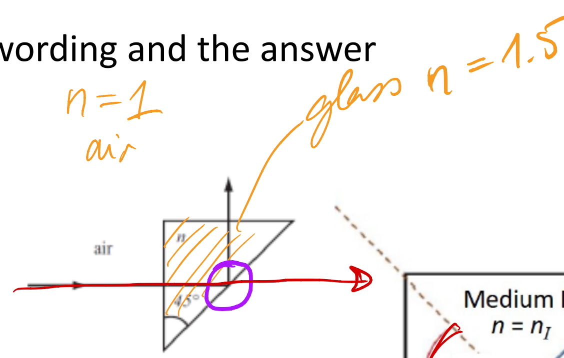
Exercise Sheet 14 :

Exercise 2 : I don't understand the drawing and how you can find him,

I don't understand the link between the wording and the answer

Exercise 2.
(Refraction and (total internal) reflection/Category II
(After training for solution: 10 min))

In the figure we sketch a situation where no light beam leaves a prism at the right edge (= total internal reflection at the second surface that the light hits). The refractive index n is such that the angle of the refracted beam at the right surface is just 90°. This is the definition of the critical angle θ_c for total internal reflection.



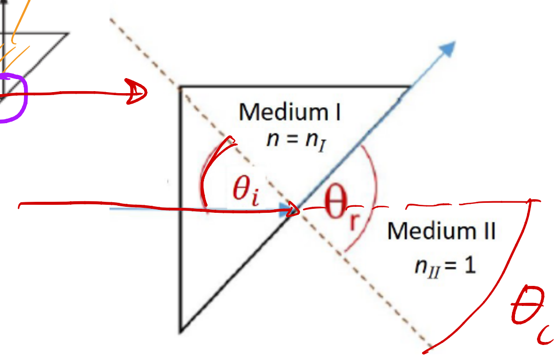
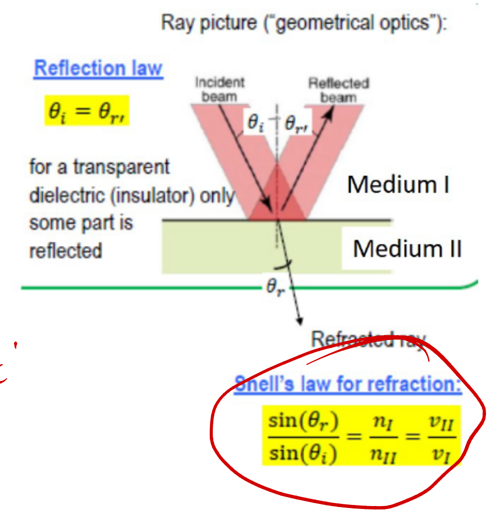


Figure 5: Sketch for the solution 4.

From lecture:



Snell's law

$$\boxed{\frac{\sin\theta_r}{\sin\theta_i} = \frac{n_I}{n_{II}}} = \frac{\sin 90^\circ}{\sin 45^\circ} \Rightarrow n_I = n = \frac{1\cdot 1}{\sin 45^\circ} = \sqrt{2},$$

7. Question

What I understand from problem 1 is that if you surround a charged conductor with a floating conductor, the ground is still considered to be at an infinite distance, so the surrounding conductor has no influence on the potential of the inner charged conductor.

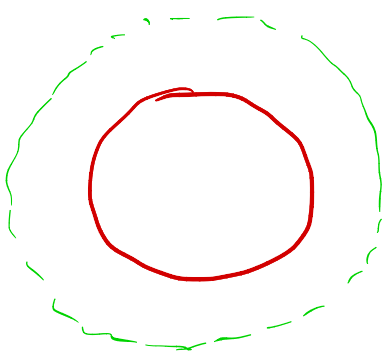
Exercise 1.

(Potential of a spherical capacitor/Category II)
An initially neutral metallic ball (conductor) of radius $R_1 = 0.1$ m is charged up with a charge Q by connecting the ball to a potential of $\phi_0 = 600$ V with an ultrathin conductor wire (with respect to the electric ground (earth) which is the potential valid at infinity). After the charging, the connection is cut abruptly such that the charges stay on the disconnected ball. Two thin hemispheres (conductors), initially uncharged, of radii $R_2 = 0.11$ m, are taken from infinity and brought in a position such that they form a closed outer spherical shell with the same center as the ball, without touching it. Calculate the potential of the inner ball for the following cases:

a) the hemispheres have no connection to the ground (earth) or the ball,

b) the hemispheres are connected to the ground (earth) using a perfect conductor,

c) the hemispheres are isolated from the ground and from the other ball when they are moved. Then, once in position, they are connected to the ball using a perfect conductor.



$\phi = \phi(\infty) = 0$

In problem 4, nothing is said about whether the outer cylinder is charged or not, so like in problem 1 I would assume that it is floating. However, as seen in your plot, the ground seems to be at the point of the outer cylinder. This situation is similar as to when we connected the ground to the outer sphere in problem 1. So why does the outer cylinder behave as if it was connected to the ground here?

Exercise 4.
(Energy density and capacitance of cylindrical coordinate/Category II; 20 mins are expected for the solution after training for the written exam)
Consider two infinitely long concentric conductor cylinders with radius a and b. They form a capacitor: The charge on the inner cylinder is $+Q$. By choosing the adequate coordinate system:

a) Find the electric field $\vec{E}$ between the two conductors as a function of the relevant charge density σ.

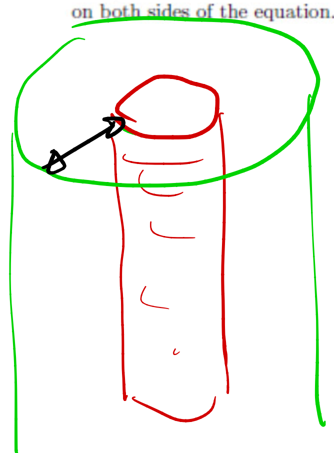
b) Calculate the potential difference between the two cylinders.

b) Due to the azimuthal symmetry of the problem, the potential does not depend on φ so: $\frac{\partial\phi}{\partial\varphi} = 0$. Since the cylinders are infinitely long and each of them represents an equipotential surface, ϕ does not depend on z and: $\frac{\partial\phi}{\partial z} = 0$. Therefore, using $\vec{E} = -\vec{\nabla}\phi$, the potential difference can be found as

$$\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \vec{\nabla}\phi\cdot d\vec{r} \tag{5}$$

Here, the direction of the line element $d\vec{r}$ can be chosen parallel or antiparallel with $\vec{E}$ but has to be consistent on both sides of the equation. Proceeding with the integration gives:

$$\begin{aligned}
\int_a^b \frac{\sigma a}{\epsilon_0 r}\hat{r}\cdot d\vec{r} &= -(\phi(b)-\phi(a)) \\
(\phi(b)-\phi(a)) &= -\frac{\sigma a}{\epsilon_0}\ln\frac{b}{a} \\
(\phi_2-\phi_1) &= -\frac{\sigma a}{\epsilon_0}\ln\frac{b}{a}
\end{aligned} \tag{6}$$

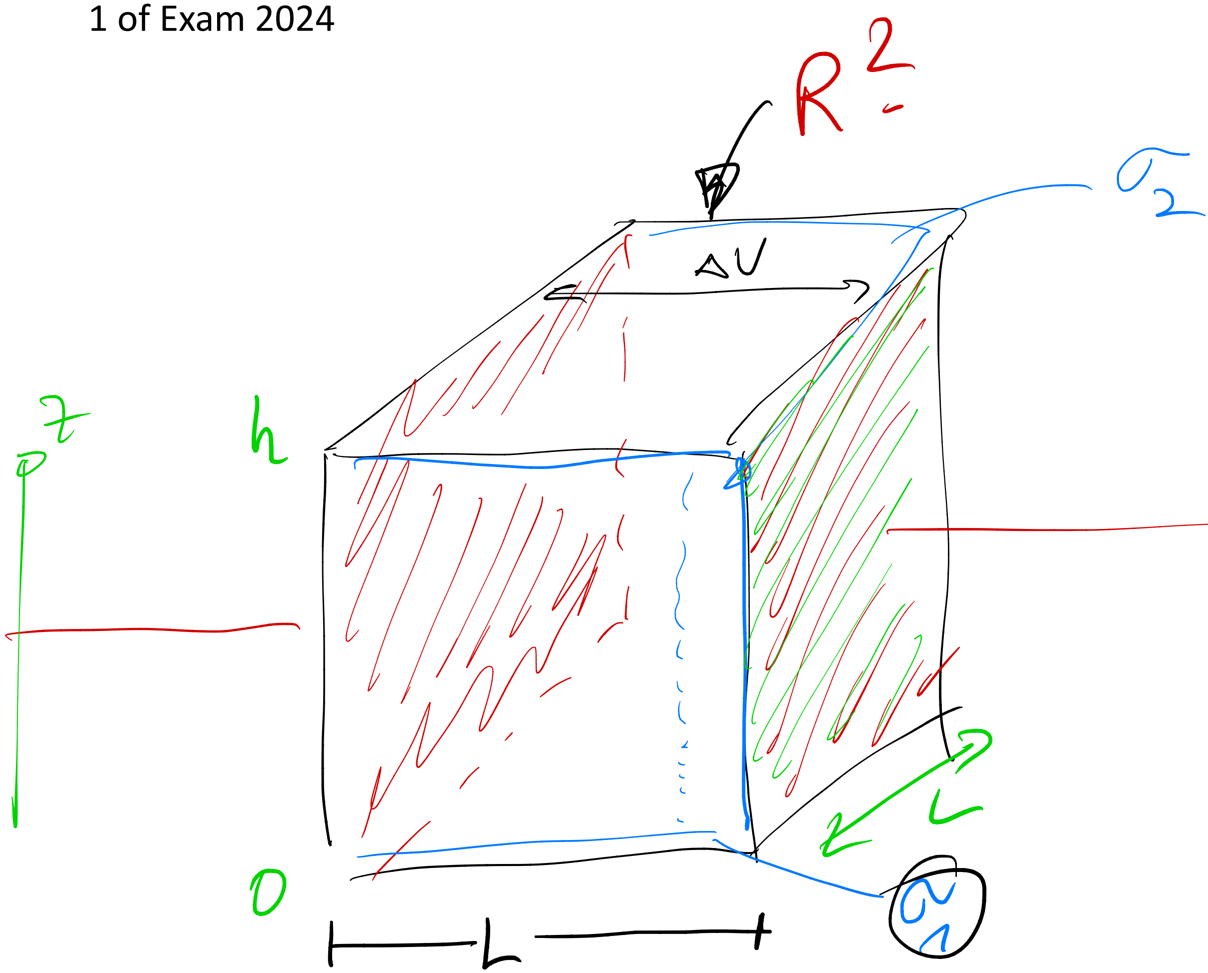


Ohm's law: $\Delta V = R \cdot I$

$$j = \sigma E = \sigma \cdot \frac{\Delta V}{L}$$

8. Question

If it is possible to explain how to process for part c) in problem 1 of Exam 2024



$$I = \iint \vec{j} \cdot d\vec{a} = \iint j\, da = \iint \sigma \cdot \frac{\Delta V}{L}\, da = L \cdot \int_0^h \left(\sigma(z)\right) \frac{\Delta V}{L} \cdot dz$$

$$\sigma(z) = \sigma_1 + (\sigma_2 - \sigma_1) \cdot \frac{z}{h}$$

$$I = (\sigma_1 + \sigma_2) \frac{h}{2} \Delta V$$

$$\Downarrow$$

$$\boxed{R = \frac{2}{h} \cdot \frac{1}{\sigma_1 + \sigma_2}}$$

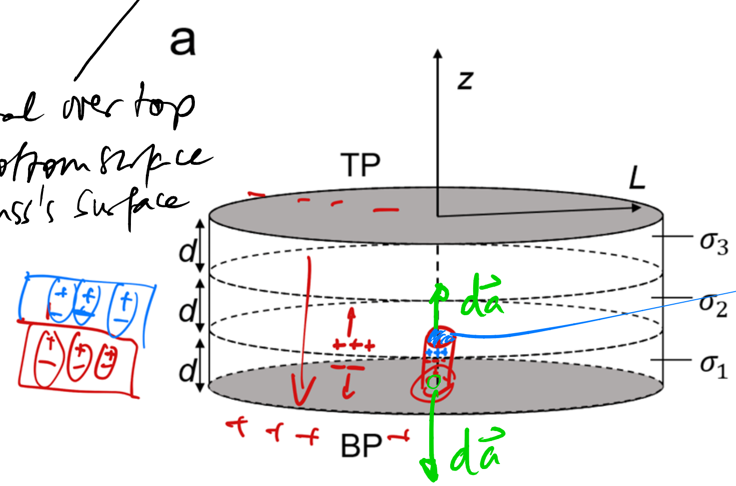
8. Question

minus sign originates from evaluation of the scalar product on top and bottom surfaces A

- Exam 2024 Problem 1: I don't understand how E_2 and E_1 are opposite. For me, they both point in the same direction.

$$\oiint \vec{E} \cdot d\vec{a} = \iint (E_2 - E_1)\, da = (E_2 - E_1)\, A = \frac{Q_{\mathrm{enc}}}{\epsilon_0} = \frac{\sigma_{\mathrm{charge},12} A}{\epsilon_0},$$

Integral over top and bottom surface of Gauss's surface



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